

5 Wk h: Yghb U\a Yb b U Wk G]hnghfY]_ j cf]gfUY`]gWk Yf 6 c hgWk UZh]b K]Yb°

I bUb[Ya Y`XYhY'8Ya c'j cf'XYf']gfUY`]gWXYb'6chgWXUZh']b K]Yb!K À\f]b['YbXYhY'a]h': YghbU\a Yb"'Dc`]nY]'`ÒghY J YfgUa a `i b['Ui Z"



K]Yb!K À\f]b[ž'² ghYffY]WX '! '5a '%\$"'* | b] '&\$&) '_Ua 'Yg']b'K]Yb!

K À\f]b['ni 'Y]bYf'i bUb[Ya Y`XYhYb'?i bX[YVi b['j cf'XYf
]gfUY`]gWX Yb'6chgWX UZhž'X]Y'gWX bY```ZØf': i fcfY'gcf[hY"'9hk U'%' "\$\$

I \f'j YfgUa a Y`hYb'g]WX 'nU\`fY]WX Y'A YbgWX Ybž'i a '[Y[Yb'U_hi Y``Y
9bhk]W_`i b[Yb'ni 'dfchYgh]YfYb"'8]Y'Dc`]nY]'gU\'g]WX'[Ynk i b[Ybž
Y]bni [fY]ZYbž'XU'X]Y'8Ya cbghfUh]cb'XYb'6Yhf]YV'XYf'6chgWXUZh
gck]Y'XYb'*bX]j]Xi U`j Yf_Y\f'Yf\YV`]WX'ghÒfhY"'8]Yg'ZØ\fhY'ni f
9bhgWXY]Xi b[ž'X]Y'J YfgUa a `i b['Ui Zni `ÒgYb"

8]Y'6Y\ÒfXYbj YfhfYhYf'XYf'@UbXYgdc`]nY]X]fY_h]cb'K]Yb _ØbX][hYb'X]Y'5i ZÒgi b['XYf'8Ya cbghfUh]cb'k]YXYf\c`h'ØVYf @Ui hgdfYW\Yf'Ub"'9]b][Y'HY]`bY\a Yf'j Yf`]Y»Yb'XUfUi Z\]b'XYb 6chgW\UZhgVYfY]W\ž'j]Y`Y'UbXYfY'YXcW\'nY][hYb'g]W\ i bbUWX []YV]['i bX'VY[UVYb'g]WX']b'Y]bYb'G]hnghfY]_'X]fY_h'j cf XYa '6chgWXUZhg[YVÀi XY"'Hfchn'XYf'5Va U\bi b[Yb'V`]YVYb'g]Y Xcfhž'k Ug'gWX`]Y»`]WX'ni '\UbXZYghYb'?cbgYei YbnYb'ZØ\fhY"

: YghbU\a Yb i bX 5bnY][Y

±0g[YgUa h'k i fXYb 'UW\h'DYfgcbYb'j cf`Ài Z]['ZYgh[Ybca a Ybž bUW\XYa 'g]Y'g]W\'k Y][YfhYbž'XYa '5i Zfi Z'ni f'5i ZÒgi b[bUW\ni _ca a Yb"'G]Y'k i fXYb']b'Y]b'Dc`]nY]Ub\U`hYnYbhfi a [YVfUW\hž'k c'g]Y'[Ya À» 'XYb'6Ygh]a a i b[Yb'XYg JYfgUa a `i b[g[YgYhnYg'Ub[YnY][h'k i fXYb"'8]Yg'\Uh']b'XYf ² ZZYbh`]W_Y]h'ZØf'8]g_i gg]cbYb'[Ygcf[hž'cV'X]Y'A U» bU\a Yb'XYf Dc`]nY]'[YfYW\hZYfh][h'k UfYb'cXYf'cV'X]Y'8Ya cbghfUbhYb']\f FYW\h'Ui Z'JYfgUa a `i b['k U\fYb'Xi fZhYb"'

8]Y'JcfZÀ`Y'k YfZYb'Ui W. Y]b'@W.h'Ui Z'X]Y'U_hi Y`Y'dc`]h]gW.Y
@U[Y']b'9i fcdUž'k c'DfchYghY'[Y[Yb'j YfgW.]YXYbY'H\Ya Yb
i bgYfYf'NY]h']a a Yf'\Ài Z][Yf'ni 'gY\Yb'g]bX'''K À\fYbX']b'K]Yb'X]Y
5i gY]bUbXYfgYhni b['a]h'XYf'Dc`]nY]'ghUhhZUbXž'[UV'Yg'nY]h['Y]W.
]b'8Yi hgW.`UbX'VYa Yf_Ybgk YfhY'8Ya cbghfUh]cbYb'[Y[Yb
FYW.hgYl hfYa]ga i g''?Ubn`Yf'C`UZ'GW.c`n`gdfUW.'Ui Z'XYa
?i fnbUW.f]W.hYbX]Ybgh'L'i bX'XUb_hY'XYb'HY]`bY\a Yfb'ZØf']\fYb
9]bgUhn']a '?Ua dZ'[Y[Yb'FUgg]ga i g'i bX'ZØf'X]Y'ZfY]\Y]h`]W.Y
; fi bXcfXbi b["'=b'?O`b'[]b[Yb'Yhk U'' \$"\$\$\$'A YbgW.Yb'Ui Z'X]Y
GhfU» Yž'i a 'Y]b'JYfVch'XYf'5Z8'ni 'ZcfXYfbž'bUW.XYa 'Y]bY
FYW.YfW.Y'XYg'A YX]i a g'7cffYWh]j 'D`AbY'ZØf'a UggYb\UZhY
5i gk Y]gi b[Yb'j cb'A YbgW.Yb'a]h'A][fUh]cbg\]bhYf[fi bX
Ui ZXYW_hY''

² ZZY b h`] W Y F Y U_h] c b Y b 'i b X Ni _i b Zhg Ui gg] W hY b

8]Y'9fY][b]ggY']b'K]Yb'i bX'X]Y'Vi bXYgk Y]hYb'DfchYghY']b
8Yi hgWX`UbX'nY][Ybž'XUgg'g]WX'j]Y`Y'A YbgWXYb'U_h]j'a]h
[YgY``gWXUZh`]WXYb'i bX'dc`]h]gWXYb'H\Ya Yb'Ui gY]bUbXYfgYhnYb"
8]Y'FYU_h]cbYb'g]bX'j]Y`ZÀ`h][.'9]b][Y'VYZØfk cfhYb'X]Y
A U» bU\a Yb'ni f'5i ZfYWXhYf\U`hi b['XYf'G]WXYf\Y]hž'k À\fYbX

 $\label{thm:linear} $$ \bXYfY'X]Y': YghbU\a Yb'U`g'9]bgW\fÅb_i b[`XYf`A Y]bi b[gZfY]\Y]h k U\fbY\a Yb''K]Y'g]W\X]Y'G]hi Uh]cb']b'XYf`Ni _i bZh'Ybhk]W_Y`b k]fXž'V`Y]Vh'UVni k UfhYb'''9]bYg'ghY\h'YXcW\'ZYgh'8]Y'6Øf[Yf]bbYb i bX'6Øf[Yf'g]bX'VYfY]hž']\fY'Gh]a a Y'ni 'Yf\YVYb'i bX'ZØf']\fY 5b`]Y[Yb'Y]bni hfYhYb''$

 $: \emptyset f ``U``Y ``6YhY]`][hYb ``V`Y]Vh`Yg`gdUbbYbX`ni ``VYcVUW\hYbž`k]Y`X]Y Dc`]h]_``Ui Z'X]YgY`J cfZÅ``Y`fYU[]YfYb`k]fX`i bX`cV`k Y]hYfY 8 Ya cbghfUh]cbYb`Zc`[Yb`k YfXYb"'\daggebY]bYf`NY]hž`]b`XYf`j]Y`Y`Y]bY _`UfY`<U`hi b[`ni `U_hi Y``Yb`H\Ya Yb`Y]bZcfXYfbž`_ÒbbYb`gc`W\Y DfchYghY`U`g`\daggebX]_Uhcf`Z\OfXUg`[YgY``gW\UZh`]W\Y'?`]a U`[Yk YfhYh k YfXYb"$

Ni gÀhn`]WX []Vh'Yg'a Y\f' \(\f \)Zcfa Uh]cbYb'ni 'XYb'i b]j YfgU`Yb 8]a Ybg]cbYb'XYf'DfchYghY']b'8Yi hgWX`UbX'Ui Z'<mark>8Yi hgWX`UbXZi b_</mark>"

8 Y hU]`g	
Cfh	K]Yb!K À\f]b[ž ^{·2} ghYffY]WX
EiY``Yb	• k k k "\Yi hY"Uh
	• k k k ") a]b"Uh
	k k k "XYi hgWX`UbXZi b_"XY

6 Y gi W Y b 'G | Y 'i b g 'U i Z 'a Y | b ! k | Y b "b Y h